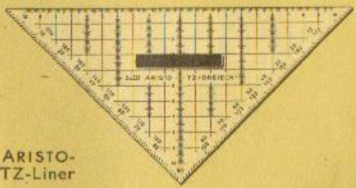
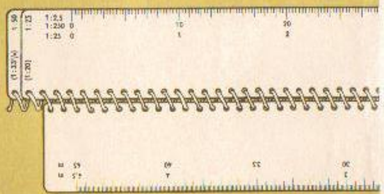
ARISTO

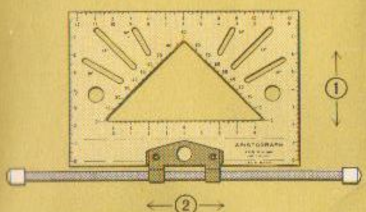


ARISTO-
TZ-Liner



ARISTO
Spiral-Scale

ARISTO TRIGON



ARISTOGRAPH

ARISTO TZ-LINER

A set square for technical drawing, combining in one instrument scales symmetrical about a centre zero, a parallel ruler and a protractor divided into 360° or 400^g.

ARISTO SPIRAL-SCALE

This consists of three 30 cm (12 in.) lengths of white ARISTOPAL, bound together with a plastics spiral. Fifteen scale ratios are displayed by means of multiple figuring on the six divided faces.

ARISTO TRIGON

A full circle protractor, divided to 360° and in radians. For setting out and measurement in either system, or for conversion from one system to the other.

ARISTOGRAPH

A drafting instrument in transparent ARISTOPAL, for quick and neat sketching, embodying a protractor divided to 180° and millimeter scales on the edges. The set square, of sides 85 × 130 mm., can be moved (1) as a parallel ruler on a roller of 200 mm length and (2) be shifted simultaneously, laterally, along the roller parallel to a given line.

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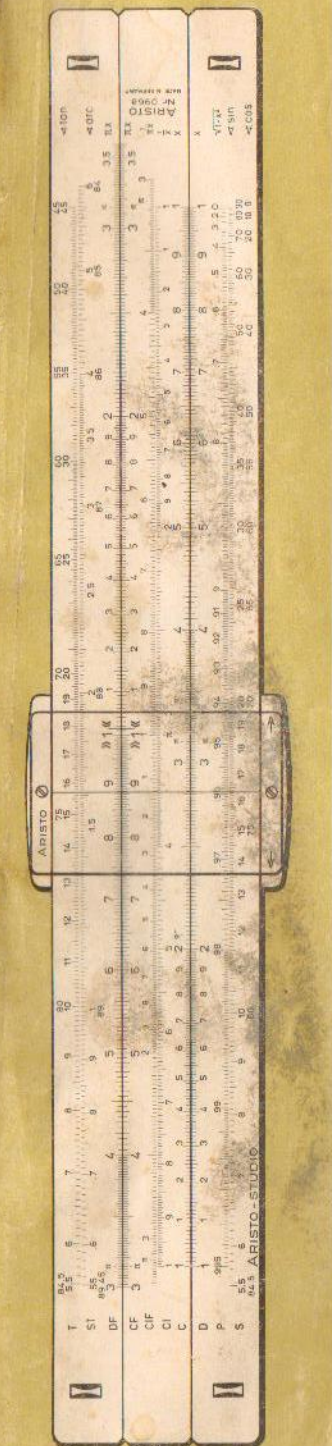


CONTENTS

THE *ARISTO* STUDIO SLIDE RULE

The ARISTO Studio is a universal Log Log slide rule for scientists, engineers and students.

1. The Scales

Trigonometric Side:

T	Scale of Tangents 5.5° to 45°; 45° to 84.5° counter-clockwise. Also available for Cotangents.	∢ tan	Upper panel of body
ST	Scale of Small Angles .55° to 6°, 84 to 89.45° counter-clockwise	∢ arc	Upper panel of body
DF	Fundamental Scale folded by π	πx	Upper panel of body
CF	Fundamental Scale folded by π	πx	On slide
CIF	Reciprocal CF Scale	$1/\pi x$	On slide
CI	Reciprocal C Scale	$1/x$	On slide
C	Fundamental Scale	x	On slide
D	Fundamental Scale	x	Lower panel of body
P	Pythagoras Scale	$\sqrt{1-x^2}$	Lower panel of body
S	Scale of Sines and Cosines 5.5° to 90°; 0° to 84.5° counter-clockwise	∢ sin, ∢ cos	Lower panel of body

Fig. 1 Trigonometric Side

Log Log Side:

LL_{01}	Log Log Scale, range: .99 — .9	$e^{-0.01x}$	Upper panel of body
LL_{02}	.91 — .35	$e^{-0.1x}$	Upper panel of body
LL_{03}	.4 — .00001	e^{-x}	Upper panel of body
A	Scale of Squares	x^2	Upper panel of body
B	Scale of Squares	x^2	On slide
L	Mantissa Scale	lg x	On slide
K	Scale of Cubes	x^3	On slide
C	Fundamental Scale	x	On slide
D	Fundamental Scale	x	Lower panel of body
LL_3	Log Log Scale, range: 2.5 — 100000	e^x	Lower panel of body
LL_2	1.1 — 3.0	$e^{0.1x}$	Lower panel of body
LL_1	1.01 — 1.11	$e^{0.01x}$	Lower panel of body

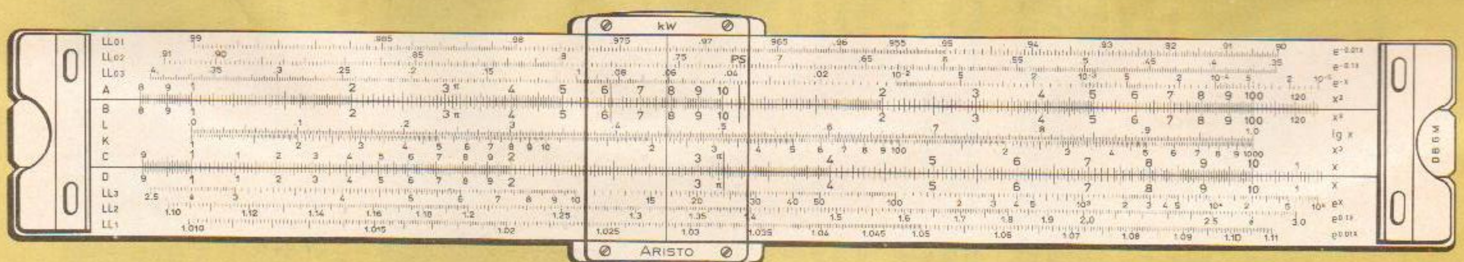


Fig. 2 Log Log Side

2. Reading the Scales

To use the slide rule efficiently for rapid calculations is essentially a matter of learning to read the scales easily and correctly.

For guidance in learning to read the scales refer to the figs. 3—6. They show the general pattern of the scales and give examples of several specific settings on the most frequently used fundamental scales C and D.

Look the C or the D scale over carefully to get a clear overall picture of the system of division governing the slide rule scales.

Fig. 3 The Main Intervals

Note that the largest, so-called primary intervals are separated by long lines labeled 1 to 10 in large numerals. These represent the first digit in the setting or reading of numbers (Fig. 3). The 10 may be given as 1 as its place is at the same time the end of one scale as well as the beginning of an imaginary second scale.

Each primary interval is again divided into ten secondary intervals. Between the primary 1 and 2 each of the ten secondary intervals is labeled with somewhat smaller numerals so that here the second digit in a number can be actually read (1—1, 1—2, 1—3 etc.), whereas in the following ranges from 2 to 10 the second digit has to be counted off.

The scale intervals diminish progressively and thus three systems of subdivision must be used for the smallest, i. e., the tertiary intervals, to avoid crowding of the lines in the region toward the end. Therefore all tertiary division lines will only be found between 1 and 2. In this first section of the scale the reading is, therefore, comparable to the reading of a rule with metric graduation, so that all numbers can here be actually read to the third digit. The fourth digits can be easily estimated as the sample settings in Fig. 4 demonstrate. Do not overlook the zero when reading the intervals immediately following the labeled marks (see 1007, 1095 in fig. 4).

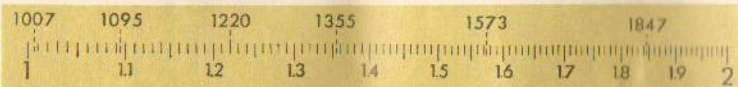


Fig. 4 Reading in the Range 1 to 2

In the next sector, between figured primary intervals 2 and 4, secondary intervals are marked but are not fully figured. Tertiary intervals are marked in units of 2 (Fig. 5). Hence, the third digit of even numbers can be read directly from the scale divisions. The third digit of odd numbers must be visually estimated. After a little practical experience you will even be able to estimate the place for the fourth digit fairly accurately.

Fig. 5 Reading in the Range 2 to 4

Between 4 and 10 the scale is kept open by marking tertiary intervals in units of 5. Only numbers, the third digit of which is 0 or 5 can be directly referred to tertiary marks. Third digits other than 0 or 5 must be located by inspection.

Fig. 6 Reading in the Range 4 to 10

The three systems of subdivision explained above are employed in all other scales and their interpretation will be no problem if you apply to them your knowledge of the fundamental scales. To avoid reading errors it is good policy to pronounce all numbers mentally digit for digit, as for instance, One-Two-Eight; not One Hundred and Twenty-eight. The reading gives no information about the decimal point and may signify any decimal variation, such as .128, 1.28, 12.8, 128 etc.

Slide rule results only supply digits in consecutive order. The decimal point is therefore at first entirely ignored and determined by a rough calculation with strongly rounded-off numbers when the computation is completed. This is a check on the order of magnitude of the result as well as an independent check on the correctness of the slide rule manipulation in a broader sense.

The calculating procedure and the required manipulations are easy to learn by thinking through and observing how a simple addition can be performed by sliding one ordinary graduated metric rule alongside a second similar rule.

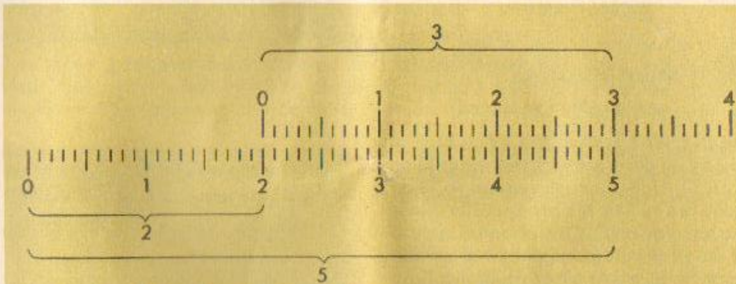


Fig. 7 Graphic Addition by Use of two Ordinary Scales

Fig. 7 shows how a mechanical addition is made with two such scales. When, for instance, the tip 0 of the uppermost scale is moved so as to coincide with the value 2 of the lower scale, we shall find the sum 5 under the value 3 of the upper scale.

Subtraction is the same process in reversed order, i. e. the length 3 of the upper scale is deducted from the total length 5 of the lower scale. It follows that, by simply setting the value 3 over 5, we can read the answer 2 under 0 of the upper scale.

Multiplication and division by use of the slide rule is exactly the same process as that described above, except that we are now dealing with logarithmic lengths, so that by adding or subtracting two segments of line we actually accomplish either a multiplication or a division. In more refined form the above discussed principle of two separate scales is embodied in the slide, movably tongued and grooved to the body of the rule. A cursor is provided to facilitate setting and reading of values to hairbreadth accuracy.

3. Explanation of Working Diagrams used in the Solution of Examples

In the following text an easily memorized method of explanation will be employed, so as to show the step-by-step operations in the respective computation with greater clarity than in the customary form of a facsimile slide rule. Parallel lines bearing their corresponding marginal labels represent the scales and the following symbols will make the diagrams very easy to interpret:

Initial setting
Each subsequent setting
Final result
Setting or reading of an intermediate result
Reverse the rule
Direction and sequence of movements
Hairline of the cursor

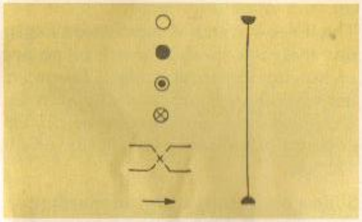


Fig. 8 shows how an example will appear in the diagram.

$$\frac{27}{6} \times 1.7 = 7.65 \text{ (Fig. 8)}$$

Intermediate result $27 \div 6 = 4.5$

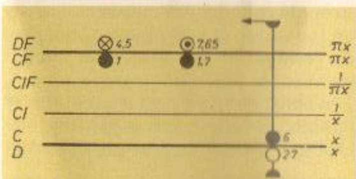


Fig. 8

4. Multiplication

(two lengths are added)

Set the left 1 (called the left index) of scale C to coincide with the value 18 on scale D. Now move the cursor to 13 on scale C and find the product 234 under the hairline on D. One essential feature of the slide rule consists in that we can perform as many other multiplications by 18 as may be required without changing the initial setting.

Fig. 9 indicates two such further operations, viz.

$18 \times .285 = 5.13$
$18 \times 7.8 = 140.4$

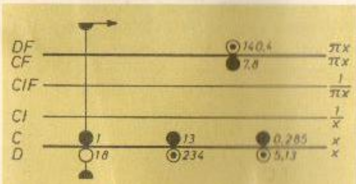


Fig. 9 $18 \times 13 = 234$
Roughly: $20 \times 10 = 200$

It will soon be observed, as in 18×7.8, that sometimes the slide projects so far beyond the end of the body scale that no reading can be taken. The simple remedy then consists in setting the right index of the slide over 18 and shifting the cursor to 7.8.

This end-for-end exchange of the indexes is rather troublesome and it can be shown that the upper pair of scales CF and DF offer an easier solution. By studying the rule you will find that the index 1 of scale CF is also matched with the value 18 of the DF scale and, therefore, we can continue reading on these scales where the lower scales break off. In fig. 9 is shown how the example $18 \times 7.8 = 140.4$ can be solved by setting the cursor to 7.8 on CF and reading the result 140.4 on DF. This procedure is always applicable provided that the slide does not project out of the body more than half its length.

It is often advantageous to make the first setting with the index of the CF scale placed opposite the multiplier on DF, because in this case there is no need to decide whether to start with the right or the left index. Furthermore, in all settings made with the upper pair of scales no more than half a slide length will ever project beyond the body scales. This means that the product can always be read on either the upper or the lower pair of scales, often on both scales simultaneously. It is advisable to repeat the previous exercises by first starting with C and D and next with CF and DF. In this way you can appreciate by experience which type of setting is the better one. (Compare chapter 9.)

5. Division

(two lengths are subtracted, multiplication in reverse)

Set the cursor over the value 2620 on scale D and draw 17.7 on C into alignment so that both values are opposite each other. The quotient 148 then appears under the left slide index.

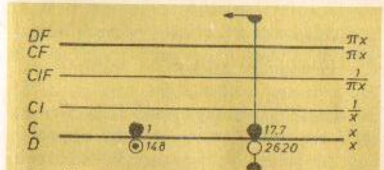


Fig. 10 $2620 \div 17.7 = 148$
Roughly: $3000 \div 20 = 150$

Notice: It is worthy of note that when this manipulation is completed the setting of the rule is also identical to that for the multiplication $148 \times 17.7 = 2620$. The only difference between multiplication and division is the order of the setting and the reading.

In the problem $582 \div 7.23 = 80.5$ the quotient will be found under the right slide index. It follows that in division no end-for-end switches of the indexes will occur. Later in this text it will be demonstrated how this feature can be usefully employed. The following chapter also contains a reference to the same subject.

The exercises in division, too, should be done, first with the C and D and then with the CF and DF scales. When the problem is set on the upper pair of scales the values are arranged in fractional notation, with the numerator above the denominator, thus:

$$\frac{3.78}{4.5} = .840$$

The answer can be read both over the index of CF on DF and under the right Index of C on D.

6. Multiplication and Division Combined

In problems of the type $\frac{a \times b}{c}$ division is usually taken first, because the slide rule is then always ready for the following multiplication.

The intermediate result of the division $345 \div 132 = 2.61$ can be ignored and the cursor moved directly to the value 22 on scale C, opposite which place the final result 57.5 appears on scale D.

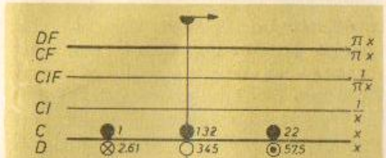


Fig. 11 $\frac{345 \times 22}{132} = 57.5$

Roughly: $\frac{300 \times 20}{100} = 60$

7. Proportions

The slide rule is particularly convenient for computations involving proportions of the shape $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$ for the reason that with one setting of the given ratio all the required terms can be obtained by simply passing the cursor along the scales. The joint between the scales of the slide and the body can be regarded as the dividing line in a common fraction.

Whenever it is possible to express a problem in the form of a proportion this type of computation should be preferred. So, for instance, the problem in fig. 11 can be easily rearranged to read: $\frac{345}{132} = \frac{57.5}{22}$. A little practice in this direction will make the slide rule user more independent of the orthodox methods of computation. It really makes no difference whether the given ratio is set $\frac{a}{b}$ or $\frac{b}{a}$ as long as the other ratios are read in conformity with the order of the first setting.

Example:

Conversion of inches to millimeters: How many millimeters are in 3, 5, 7 inches? The initial ratio is known to be 1 in = 25.4 mm. Hence, expressing our problem in proportional form, we write:

$$\frac{1}{25.4} = \frac{3}{x} = \frac{5}{y} = \frac{7}{z}$$

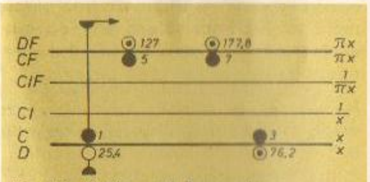


Fig. 12 Ratios and Proportions

Once the given ratio 1 : 25.4 is set on the rule the other terms can be found by merely moving the cursor as required: x = 76.2 mm, y = 127.0 mm and z = 177.8 mm.

8. The Reciprocal Scales CI and CIF

The scales are the counterparts of their respective companions C and CF, except that their graduations and numerations run in the opposite direction. By virtue of this arrangement the reciprocal $\frac{1}{x}$ of any value x on the fundamental scale C can be directly read on CI under the cursor line. The utility of this feature consists in being able to convert at will a multiplication to a division and vice versa.

So, for instance: $4 \times 5 = 4 \div \frac{1}{5}$ and $4 \div 5 = 4 \times \frac{1}{5}$.

Expressions of the form $a \times b \times c$ or $\frac{a}{b \times c \times d}$ are solved by alternate multiplication and division. In the course of a computation we may freely switch from the group of scales C, D and CI to the group CF, DF and CIF to avoid resetting the slide in multiplication. Fig. 13 shows such a situation.

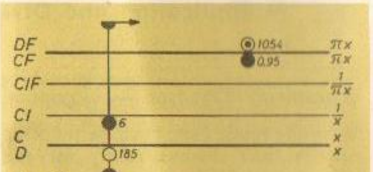


Fig. 13

185 × 6 × .95 = 1054
Roughly: 200 × 6 × 1 = 1200

Rearrange the problem to read: $\frac{185}{1/6} \times .95$. Use the cursor hairline to bring 185 on scale D into coincidence with 6 on the reciprocal scale CI, as in any ordinary division. The intermediate result 1110 on D under the index of C can be ignored. The subsequent multiplication by .95 is best performed by use of scale CF and the product 1054 is then read directly over .95 on scale DF.

9. The Folded Scales CF and DF

We have already had occasion to use these scales in multiplication and division, in which class of computation they are principally employed (See chapters 4 and 5). The folded scales are identical to the fundamental scales, except that any location on one type of scale is laterally displaced by the value $\pi = 3.14159$ relative to the other scale. This means that the value π on the folded scales is exactly matched with the indexes of the fundamental scales. Consequently the graduation runs from π past 1, in about the middle of the rule, to π at the right extreme. There are short extensions of the graduations at both ends for greater convenience.

The index 1 of the CF scale is always aligned to the same value on DF to which the index 1 of C is aligned on D. It follows that we can also begin a calculation with the CF/DF scales and with advantage, too. There is no more need to decide between two indexes for the first setting.

The two scale pairs CF/DF and C/D form a team working in perfect unison. Whenever the point is reached where one pair fails to give the answer, the other pair takes over. No more "resetting the slide".

The yellow strips on the slide serve as a precaution against errors, since the slide scale C moves over D but CF moves under DF.

9.1 Tabulations avoiding Resetting the Slide

$$y = 29\,x$$

x	1.7	3.45	5.0	10
y	49.3	100	145	290

For x = 5 the upper pair of scales CF and DF provides the answer without resetting the slide.

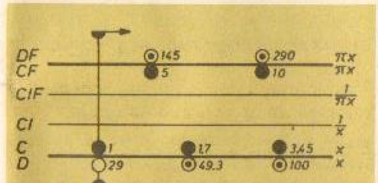


Fig. 14

$$y = \frac{28.2}{x} = 28.2 \times \frac{1}{x}$$

x	7.43	2.92	1.567
y	3.795	9.66	18.0

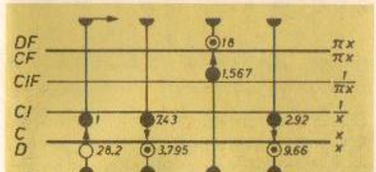


Fig. 15

$$y = \frac{x}{18.2} = \frac{1}{18.2} \times x$$

x	3.17	112.1
y	.1742	6.16

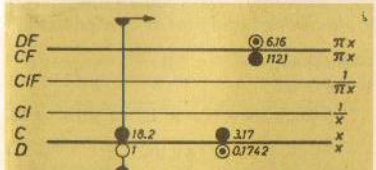


Fig. 16

9.2 Direct Reading of Multiplications and Divisions involving π

Another advantage of this folded arrangement consists in simplifying various computations involving the factor π. It will be clear that any switch-over from D to DF automatically supplies the product of any number set on D multiplied by the factor π. Conversely then the division by π is achieved by following the opposite course.

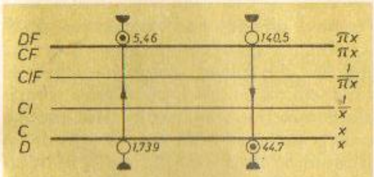


Fig. 17 $x\pi$ and x/π

Typical Problems:

Circumference of Circles $C = d\pi$
Angular Velocity $\omega = 2f\pi$
Area of Circles $A = r^2\pi$

The first two formulas can be computed with one cursor setting, whereas we must first perform the multiplication $r \times r$ with the scales C and D to find the circle area (see also chap. 16.2).

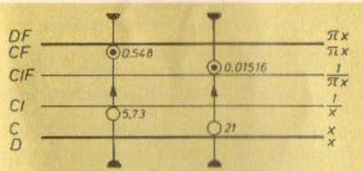


Fig. 18 $\frac{\pi}{x}$ and $\frac{1}{x\pi}$

$1.739\pi = 5.46 \qquad \frac{140.5}{\pi} = 44.7$

$\frac{\pi}{5.73} = .548 \qquad \frac{1}{21 \times \pi} = .01516$

10. The Scales A, B and K

When the cursor hairline is set to any value of x on scale C, x^2 can be read on scale B and x^3 on scale K. In the opposite order the switch-over from K to C furnishes the cube root and from B to C the square root.

a) $2^2 = 4 \qquad 2^3 = 8$

b) $3.27^2 = 10.7 \qquad 3.27^3 = 35$

c) $\sqrt[2]{9} = 3 \qquad \sqrt[3]{27} = 3$

d) $\sqrt[2]{51} = 7.14 \qquad \sqrt[3]{364} = 7.14$

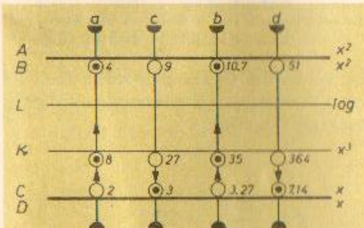


Fig. 19 Powers and Roots

When it is clear that the root concerned will fall within the range 1 to 10 no special calculating rules need to be applied. This is the case when the radicand of a square root lies somewhere between 1 and 100 or the radicand of a cube root between 1 and 1000. In all other cases it is necessary to reduce the radicand to the more handy form of a power of ten notation.

Exercises:

$$\sqrt[2]{3200} = \sqrt[2]{100 \times 32} = \sqrt[2]{10^2 \times 32} = 10 \times \sqrt[2]{32} = 10 \times 5.66 = 56.6$$

$$\sqrt[3]{.1813} = \sqrt[3]{\frac{181.3}{1000}} = \frac{1}{10} \times \sqrt[3]{181.3} = \frac{1}{10} \times 5.66 = .566$$

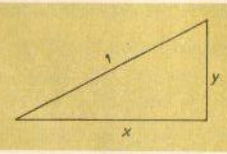
Multiplication and division can also be done by using the scales A and B by the same process as that used for C and D, but the precision obtained will be somewhat less refined. In many problems beginning with a squaring operation it is an advantage to be able to continue the computation on the A and B scales.

It is recommended to repeat the examples given in the chapters 4 to 7 with the scales of squares A and B for practice in the use of these scales and in order to judge the accuracy here obtainable as compared with that of the fundamental scales.

11. The Pythagoras Scale P

For a right triangle having the hypotenuse 1 we have the following relation: $y = \sqrt{1 - x^2}$.

Fig. 20



For any setting of x on the fundamental scale D we find the value y on scale P and inversely $x = \sqrt{1 - y^2}$ on D, when y is set on P.

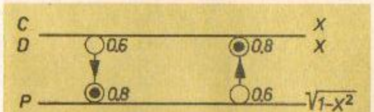
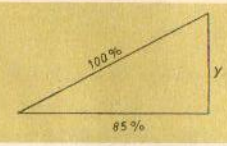


Fig. 21

Example: $\sqrt{1 - .6^2} = .8$; $\sqrt{1 - .8^2} = .6$

Always choose your settings and readings where the highest degree of accuracy may be expected, for instance $\sqrt{1 - .15^2} = .9887$, here .15 is set on scale D.

Fig. 22



Example in electrical engineering:
Apparent load = 100%
Effective load = 85%
Wattless load $= \sqrt{1 - .85^2} = .527$, therefore 52.7%.

The Pythagoras Scale may be similarly applied in such cases where the hypotenuse is either .01. 1 or 100 etc. especially in changing from sine to cosine (e. g. $\sin\alpha = \sqrt{1 - \cos^2\alpha}$). For all problems reducible to right angles, e. g. work with vectors, complex numbers, coordinates etc. the trigonometric method is more elegant. (See chapter 13.)

When the number whose root is wanted is near .01, 1 or 100 etc., greater accuracy is obtained by converting, for example: $\sqrt{.95} = \sqrt{1 - .05} = .9747$ (.05 is set on scale A). This method brings a more accurate solution for roots greater than $\sqrt{0.65}$.

12. Trigonometric Functions

When the hairline is set to the angle on scale S, T or ST, the respective function can be read on the D scale. By reversing the process we obtain the angle corresponding to the given function. Angles are given in degrees divided decimally.

The slide rule can only supply the functions of angles in the first quadrant directly. Use the following table to reduce angles of other quadrants to the first:

	$\pm\alpha$	$90^\circ \pm \alpha$	$180^\circ \pm \alpha$	$270^\circ \pm \alpha$
sin	$\pm \sin\alpha$	$+ \cos\alpha$	$\mp \sin\alpha$	$- \cos\alpha$
cos	$+ \cos\alpha$	$\mp \sin\alpha$	$- \cos\alpha$	$\pm \sin\alpha$
tan	$\pm \tan\alpha$	$\mp \cot\alpha$	$\pm \tan\alpha$	$\mp \cot\alpha$
cot	$\pm \cot\alpha$	$\mp \tan\alpha$	$\pm \cot\alpha$	$\mp \tan\alpha$

12.1 The Sine Scale S extends from 5.5° to 90° and also contains the cosine values in red numerals (0° to 84.5°), running backwards from right to left. All sines and cosines read on the fundamental scale must be given the prefix "0"...

The sines of angles $45^\circ < \alpha < 90^\circ$ are more accurately readable on the wider spaced red P scale, here applicable in virtue of $\sin\alpha = \sqrt{1-\cos^2\alpha}$. The angle is in this case set on the red-figured cosine graduation of the S scale.

Colour rule for sines: All settings and readings in like colours. Since $\cos\alpha = \sqrt{1-\sin^2\alpha}$, the circumstances are analogous for the cosines of angles $5.5^\circ < \alpha < 45^\circ$, and the colour rule to be observed is: For any setting made on the S scale, in the one colour, read the function in the other colour on either D or P, as the case may be.

$\sin 26^\circ = .438$

$\sin 82^\circ = \sqrt{1-\cos^2 82^\circ} = .9903$

$\text{arc sin } .54 = 32.7^\circ.$

Always read **like** colours!

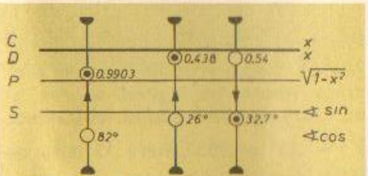


Fig. 23 Sines

$\cos 75^\circ = .2588$

$\cos 187^\circ = -\cos 7^\circ = -\sqrt{1-\sin^2 7^\circ} = -.99255$

$\text{arc cos } .9852 = 9.87^\circ$

Always read **unlike** colours!

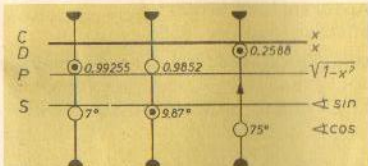


Fig. 24 Cosines

12.2 The Tangent Scale T has black numbers for angles from 5.5° to 45° and in opposite progression from 45° to 84.5° in red colour. Since the tangent of any angle $\alpha < 45^\circ$ is always smaller than 1, the corresponding functional values on the D scale take the prefix "0". ...

For angles $\alpha > 45^\circ$ apply the red numbers and obtain the function from the red numbers of the CI scale in consideration of $\tan\alpha = 1/\tan(90^\circ - \alpha)$. These values are always larger than 1. In this operation make sure that the slide and body scales are accurately matched. Set and read tangents in matching colours. Since $\cot\alpha = 1/\tan\alpha$ the cotangents of angles $\alpha > 45^\circ$ are found on the D scale and those of $\alpha < 45^\circ$ on the CI scale.

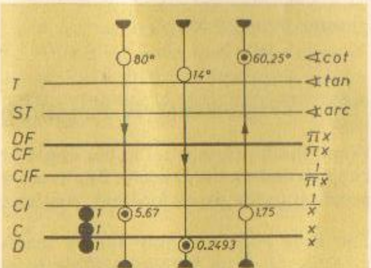


Fig. 25 Tangents

Therefore, switch colours between setting and reading.

$\tan 14^\circ = .2493$

$\tan 80^\circ = \dfrac{1}{\tan 10^\circ} = 5.67$

$\tan 80^\circ = \cot 10^\circ = 5.67$

$\text{arc tan } 1.75 = 60.25^\circ$

$\cot 77^\circ = .2309$

$\cot 9^\circ = \dfrac{1}{\tan 9^\circ} = 6.31$

$\text{arc cot } 2.0 = 26.57^\circ$

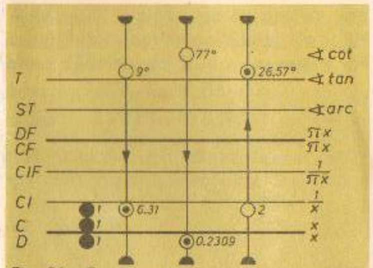


Fig. 26 Cotangents

12.3 The ST Scale (Small Angles · Large Angles · Degrees ← → Radians)

This scale includes angles between .55° and 6° and is so designed that for any angle α set on its graduation the D scale gives the radian measure $\frac{\pi}{180} \times \alpha$ of this angle.

For practical purposes the sines, the tangents and the radian measure of angles smaller than 6° are so nearly alike, that the ST scale serves equally for all three functions. Conversely, too, for the cosines and cotangents of angles larger than 84.5° which are not included in the S and T scales, using the complementary angle in this case. Expressing the foregoing in formula form we have:

$$\sin\alpha \approx \tan\alpha \approx \cos(90^\circ - \alpha) \approx \cot(90^\circ - \alpha) \approx \frac{\pi}{180}\alpha = .01745\,\alpha \text{ rad}$$

Examples: $\sin 1^\circ \approx \tan 1^\circ \approx \cos 89^\circ \approx \cot 89^\circ \approx \frac{\pi}{180} = .01745$ rad (on D)

$$\cot 1^\circ \approx \frac{1}{\tan 1^\circ} \approx \frac{180^\circ}{\pi} = 57.3 \text{ (on CI)}$$

$\sin 3.8^\circ = .0663,\quad \cos 88.2^\circ = .0314,\quad \tan 2.74^\circ = .0478.$

The cosines of small angles and, conversely, the sines of large angles cannot be found directly with the slide rule because the limited interval between 80° and 90° (0° and 10° in red) of the S scale makes it impossible to place the angle accurately. When these are involved the solution requires the use of the part of a series progression:

$$\cos\alpha = 1 - \frac{\alpha^2}{2} \text{ (}\alpha\text{ in radians)}$$

$$\cos 1^\circ = 1 - \frac{.01745^2}{2} = 1 - .000152 = .999848.$$

Note that the required square of any given angle set on the ST scale is directly readable in radians on the A scale. To find the angle corresponding to a given cosine reverse the process.

12.4 Conversion of Degrees to Radians (and vice versa)

Scale ST is a duplicate of the fundamental scale, modified only in so far as the one graduation is displaced laterally by the value $\frac{\pi}{180}$ relative to the other graduation. Therefore by following the cursor line from scale ST to D we achieve the conversion of degrees to radians, and vice versa. This form of calculation is applicable not only to the small angles discussed above but to large angles as well, by virtue of the decimal subdivision of the degrees, in consideration of the fact that the displacement of the graduations by $\frac{\pi}{180}$ is simply a constant multiplication factor.

Any setting of an angle α may also be regarded as representing $.1\,\alpha$, $10\,\alpha$, $100\,\alpha$ etc. and the decimal point in the radian is then placed accordingly.

For instance:

$.1^\circ = .001745$ radians		$.3^\circ = .00524$ radians
$10.0^\circ = .1745$ radians		$3.0^\circ = .0524$ radians
$100.0^\circ = 1.745$ radians		$30.0^\circ = .524$ radians

When small angles are given in terms of minutes or seconds these must be converted to decimal parts of the degree as follows:

$$1' = \frac{1^\circ}{60} \text{ and } 1'' = \frac{1^\circ}{3600}.$$

12.5 The Gauge Marks for Minutes ′ and Seconds ″

The marks ϱ' and ϱ'' on the C scale give a means of direct computation of radian equivalents when the angle is given in minutes or seconds. They represent the conversion factors:

$$\varrho' = \frac{180}{\pi} \times 60 = 3438 \text{ for minutes and } \varrho'' = \frac{180}{\pi} \times 60 \times 60 = 206\,265 \text{ for seconds.}$$

Hence: α in radians $= \frac{\alpha'}{\varrho'} = \frac{\alpha''}{\varrho''}$

In reversed order: $\alpha' = \alpha \text{ rad} \times \varrho'$

$\alpha'' = \alpha \text{ rad} \times \varrho''$

Examples:

$$22' = \frac{22}{3438} = .00640 \text{ radians } \left(\text{Roughly } \frac{20}{3000} = .006\right)$$

$$380'' = \frac{380}{206265} = .001843 \text{ radians } \left(\text{Roughly } \frac{400}{200000} = .002\right)$$

$.0045$ radians $= .0045 \times 3438 = 15.8'$. (Roughly $.005 \times 3000 = 15$)

The gauge marks for minutes and seconds are a useful aid in solving circle sectors for radii r, arc lengths b and central angles α.

$$\alpha = \frac{b}{r} \times \varrho \qquad b = \frac{\alpha \times r}{\varrho}$$

Examples:

$$\alpha = \frac{.6}{45} \times \varrho' = 45.8'$$

$$b = \frac{48'' \times 67}{\varrho''} = .0156$$

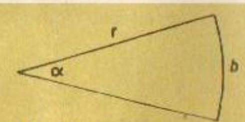


Fig. 27

13. Trigonometric Solution of Plane Triangles

The law of sines is a convincing example of the efficiency of the slide rule in solving proportions:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

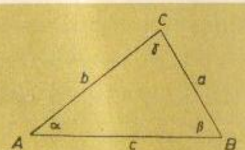


Fig. 28

When, for example, the angle α on the S scale and the side opposite this angle on the C scale are brought to a match, all other parts of the triangle are instantly readable by cursor movement.

In practice this form of computation is mostly concerned with right triangles. In this case we have $\gamma = 90^\circ$, hence $\sin\gamma = 1$. Since $\alpha = 90^\circ - \beta$ and $\beta = 90^\circ - \alpha$ the law of sines is therefore rearranged to the formula:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{1} = \frac{a}{\cos\beta} = \frac{b}{\cos\alpha}$$

and further: $\tan\alpha = \frac{a}{b}$

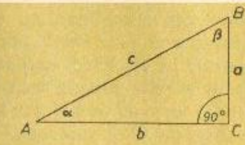


Fig. 29

Depending on the given elements there are two basic operations, viz.

A. Given: Any two parts (except the case B).

B. Given: The small sides a and b.

Example of A:

Given: c = 5, a = 3

Required: α, β, b.

Remember that: $\beta = 90^\circ - \alpha$

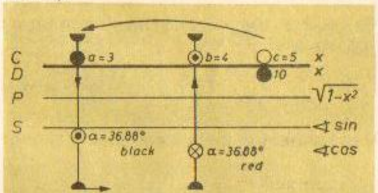


Fig. 30 Given the Hypotenuse

Set c = 5 on the C scale opposite the right index of D. Shift the cursor to a = 3 on the C scale and read the angle $\alpha = 36.88^\circ$ on the S scale. Leaving the slide undisturbed, shift the cursor to $\alpha = 36.88^\circ$ (red) or $\beta = 90^\circ - \alpha = 53.12^\circ$ on the same scale and read the side opposite the angle, b = 4, on the C scale. All variations of this problem are solved by similar procedures, except when the given elements are the two legs. In this case proceed as follows:

Example of B:

Given: a = 3, b = 4

Required: α, β, c

$$\tan\alpha = \frac{3}{4} = 3 \times \frac{1}{4}$$

$$\alpha = 36.88^\circ$$

$$c = \frac{a}{\sin\alpha} = \frac{3}{\sin 36.88^\circ} = 5$$

After finding α on scale T, as shown in fig. 31, the cursor is set over the same angle on scale S with slide unchanged. Now read c = 5 under the hairline on scale CI.

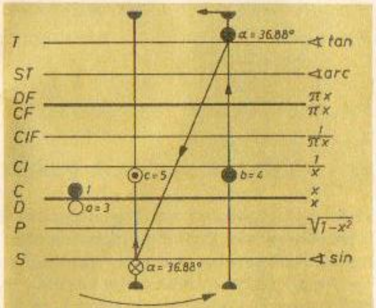


Fig. 31 Given the two Sides

Another example of the same kind of calculation:

Given: a = 15, b = 25

Solution: $\alpha = 30.96^\circ$ $\beta = 90^\circ - 30.96^\circ = 59.04^\circ$ c = 29.16

The two classes of solutions for right triangles discussed in the preceding text are of particular significance in problems involving coordinates, vectors, and complex numbers. Such problems invariably require conversions of rectangular coordinates to polar coordinates and vice versa.

When a > b, so that $\alpha > 45^\circ$, the procedure remains unchanged: Always set the smaller side on D and the greater side on CI. In this case the angle α is read as complementary angle by use of the red numeration of the T scale and, likewise, set α within the red numeration of the S scale.

A complex number can be written:
$Z = a + j\,b$ (component form) or
$Z = r \times e^{j\varphi} = r/\underline{\varphi}$ (exponential form)

Conversions from component to exponential form and vice versa occur quite frequently in electrical engineering problems for the reason that, in the component form $Z = a + j\,b$, the values are easy to add or subtract. The exponential form $Z = r/\underline{\varphi}$ is equally convenient for performing multiplications and divisions as well as in finding roots and powers.

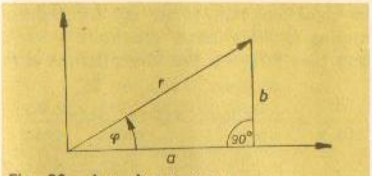


Fig. 32 $\Delta x, \Delta y \longleftrightarrow r, \varphi$

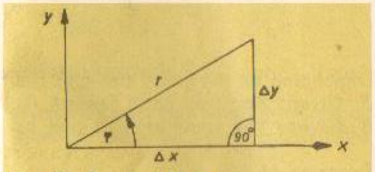


Fig. 33 $Z = a + j\,b = r/\underline{\varphi}$

Examples for converting complex numbers:

$Z = 4.5 + j\,1.3 = 4.68\,/\underline{16.13^\circ}$

$Z = 6.7\,/\underline{49^\circ} = 4.39 + j\,5.05$

The solution of these examples is shown in the explanations above and in fig. 33.

14. The Log Log Scales LL1—LL3 and LL01—LL03

All Log Log scales are used together with the fundamental scales C and D. The three e^{+x}-scales LL1, LL2 and LL3 cover the range 1.01 to 100,000 and the three e^{-x}-scales LL01, LL02 and LL03 the range .00001 to .99.

The e^{+x} and e^{x-} -scales are reciprocal to each other. Reciprocals of numbers < 2.5 can be read with a better degree of refinement than can be expected when using the scales CI or CIF, e. g., the reciprocal value of 1.0170 is .98328.

Attention: The Log Log scales supply immutable values, either whole numbers or numbers with their fractional parts in decimals. This means that when we read 1.35, this is the exclusive value concerned. It is not decimally variable as are values found on the fundamental scales.

14.1 The 10th and 100th Powers and Roots

The Log Log scales are mutually coordinated in such a manner that, in passing from one scale to the adjacent scale, the tenth power or the tenth root of a number set on the one scale can be read on the neighbouring scale, depending on the direction in which the reading is made. The examples depicted in fig. 34 will make it clear how the tenth and hundreth power or root of a given number can be determined by the simple process of following the cursor hairline to the appropriate scale.

Examples:

$1.015^{10} = 1.1605$

$1.015^{100} = 4.43$

$1.015^{-100} = .2257 = 1/4.43$

$1.015^{-10} = .8617 = 1/1.1605$

$1.015^{-1} = .98522 = 1/1.015$

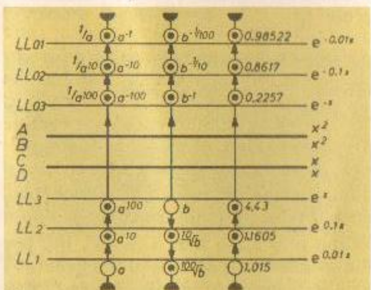


Fig. 34 The LL Scale System

Variations of readings in the same range of numbers:

$\sqrt[10]{4.43} = 1.1605 \qquad .98522^{100} = .2257$

$\sqrt[100]{.2257} = .98522 \qquad \dfrac{1}{\sqrt[10]{4.43}} = .8617$

Problems such as the above will hardly ever arise in practice but are useful to gain a clear insight into the system of Log Log scales.

14.2 Powers $y = a^x$

Raising a number to any power is done exactly as multiplications are performed with the fundamental scales.

Procedure:

a. Use the cursor to set the index of scale C to the base "a" on the appropriate LL scale.

b. Shift the cursor hairline to the value of the exponent on C.

c. Read the power y under the hairline on the corresponding LL scale. (See below: Reading Rules.)

When the slide is set to the value of the base "a" we obtain a complete table of values corresponding to the function $y = a^x$. Fig. 35 depicts the setting of the slide to be made for the function $y = 3.2^x$ showing the cursor aligned to the exponent 2.5 and its decimal variates.

Examples:

		Reading on scale
$3.2^{2.5}$	= 18.3	LL3
$3.2^{.25}$	= 1.338	LL2
$3.2^{.025}$	= 1.0295	LL1
$3.2^{-2.5}$	= .0546	LL03
$3.2^{-.25}$	= .7476	LL02
$3.2^{-.025}$	= .97134	LL01
$3.2^{3.1}$	= 36.8	LL3
$3.2^{.36}$	= 1.520	LL2

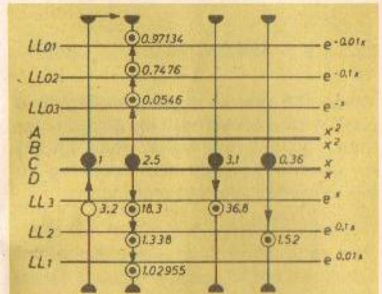


Fig. 35 Powers

Reading Rules:

a. For positive exponents set and read in the same group of scales LL1—LL3 or LL01—LL03 i. e. use scales having numerals of uniform colour. For negative exponents we must switch over from one group of scales to the other (alternating the colours).

b. In conformity with the labels given at the right end of each scale, read on the adjoining scale with the inferior label for each place that the decimal point in the exponent is moved to the left. (Cf. example fig. 35.)

c. When the base is set with the right slide index, all readings must be taken from the adjoining "higher" labeled scale. (Cf. example fig. 38).

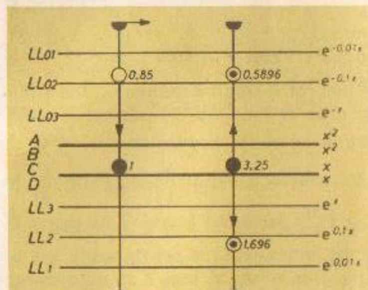


Fig. 36 Powers of Numbers < 1

For 0 < a < 1 find the powers with positive exponents on the three

Log Log scales LL01—LL03 and those with negative exponents on the three Log Log scales LL1—LL3.

$.85^{3.25} = .5896$
$.85^{-3.25} = 1.696$ } Fig. 36

$1.46^{2.7} = 2.78$
$1.46^{-2.7} = .36$
$.685^{2.7} = .36$
$.685^{-2.7} = 2.78$ } Fig. 37 and Fig. 38

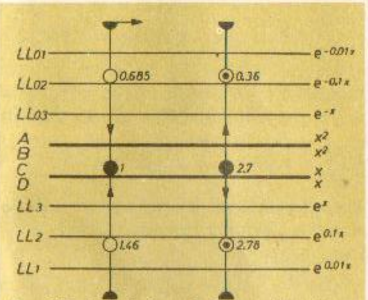


Fig. 37 Left Index of C over Number

14.3 Exceptional Cases of $y = a^x$

Since the range of the Log Log scales is restricted, cases will arise where the exponent or the base is either too great or too small to permit direct reading of the power.

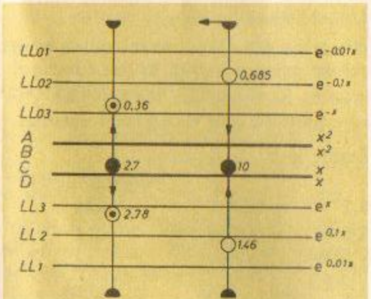


Fig. 38 Right Index of C over Number

14.3.1 $y > 100{,}000$ and $y < .00001$

When the power corresponding to a base with a large exponent is greater than 100,000 or smaller than .00001 the alternative consists in breaking up the exponent into several factors.

Example:

$$3.14^{19} = 3.14^{6+6+7} = (3.14^6)^2 \times 3.14^7 = .955^2 \times 10^6 \times 3.02 \times 10^3 = 2.76 \times 10^9$$

For expressions with negative exponents the procedure is, of course, analogous.

14.3.2 $.99 < y < 1.01$

When for a small exponent x, the value of the power is either smaller than 1.01 or greater than .99 the answer is obtained by use of an approximation.

From the series expansion

$$a^{\pm x} = 1 \pm \frac{x}{1!} \log_e a + \frac{x^2}{2!} \left(\log_e a\right)^2 \pm \frac{x^3}{3!} \left(\log_e a\right)^3 + \ldots$$

can be derived $a^{\pm x} \approx 1 \pm x \log_e a$ for $|x \log_e a| \ll 1$.

If the index of C is set opposite the base value a on LL by aid of the cursor line, the value of $\log_e a$ is automatically set on the D scale (See chap. 14.6) so that the multiplication by x can be performed by cursor movement along the C scale and then reading $x \times \log_e a$ on D. The product added to 1 or.

respectively, subtracted from 1 gives the power $a^{\pm x}$. The smaller the exponent the more precise will be the result.

Returning to our previous example with the base 3.2 (Fig. 35), we can now continue, e. g.:

Examples: $3.2^{.0025} \approx 1 + \log_e 3.2 \times .0025$

$\approx 1 + .002908 = 1.002908$

$3.2^{-.0025} = 1 - .002908 = .997092$

When the exponent is still further reduced through shifts in the location of the decimal point, the answer only varies in respect of the number of ciphers or nines immediately following the decimal point.

$$3.2^{.00025} = 1.0002908$$

14.3.3 $.99 < a < 1.01$

When, in the power $y = a^x$, the base exceeds .99 but is less than 1.01 the solution is again obtained by approximation, as follows:

In accordance with the series expansion applied to the case reviewed in the preceding paragraph: $a^{\pm x} \approx 1 \pm x \times \log_e a$. Since a, in the present case, is near 1 we can write $a = 1 \pm n$, from which we can further derive:

$$a^x = (1 \pm n)^x \approx 1 + x \log_e (1 \pm n) \qquad \log_e (1 \pm n) \approx \pm n \text{ (for } |n| \ll 1)$$

$$(1 \pm n)^x \approx 1 \pm nx \text{ (for } |nx| \ll 1) \quad (1 \pm n)^{-x} \approx 1 \mp nx \text{ (for } |nx| \ll 1)$$

It is immaterial whether, in the range here discussed, $\log_e (1 \pm n)$ is set in scale LL or the value n in scale D, in consideration of $\log_e (1 \pm n) \approx \pm n$. The smaller the magnitude of the value n the closer the correctness of the approximation. It follows, then, that where the LL scale breaks off scale D can be used as the continuation of the LL scale, in this case substituting $\pm n$ for $1 \pm n$. When the index of the C scale coincides with n on the D scale this setting is practically identical with the setting $\log_e (1 \pm n)$ within an imaginary additional LL scale covering the range 1.001 to 1.01 or, respectively, .990 to .999 and so on. The computation then continues by looking up the power as previously discussed. Actually any answer read on the D scale is derived from a simple multiplication but has to be complemented by the addition of "1" or by subtraction from "1", as the case may be. When, with growing exponent, the power falls within the readable range of the LL scales readings are taken directly from these scales.

Examples:

$1.0023^{3.7} = (1 + .0023)^{3.7} = 1.00851$	Set on scale D, read on scale D and add 1
$1.0023^{37} = 1.0888$	Set on scale D, read on scale LL1
$.9977^{3.7} = (1 - .0023)^{3.7} = .99149$	Set on scale D, read on scale D and deduct from 1
$.9977^{37} = .9184$	Set on scale D, read on scale LL01

With the cursor hair aligned over the left index of D, the amount of displacement relative to the line for 1.01 on LL1 provides a good check on the amount of error in the approximative computation. The maximum degree of error will be introduced into the approximation when both setting and reading take place on scale D in substitution for the Log Log scales.

14.3.4 Improving the Accuracy

The precision can be improved when the disparity between reading on the D scale and the actual Log Log scale within the range 1.001 to 1.01 is corrected by also applying both the linear and the quadrature term in the series expansion to the previously discussed procedure.

A. $\log_e(1 \pm n) \approx \pm n\,(1 \mp n/2)$ for settings of the base on D

B. $e^{\pm x} \approx 1 \pm x\,(1 \pm x/2)$ for readings taken from D

When the result is obtained from a Log Log scale, only formula A need be applied before making the setting on scale D. If, however, scale D is used exclusively in a computation, corrections have to be applied to the setting as well as to the answer (formula B).

Example:

$1.0023^{3.7} = 1.00854$

For $n = .0023$ substitute the setting
$.0023\,(1 - 1/2 \times .0023) = .0023 \times .99885 = .002297$ by slide index on scale D.

The operation required to determine the 3.7th power, viz. $1 + .002297 \times 3.7$, gives 1.00850. This reading, because of its taking place on scale D, requires correction by formula B, as follows:

$$.00850\,(1 + 1/2 \times .00850) = 0.00850 \times 1.00425 = .00854$$

After adding the "1", the final answer then is 1.00854 (exactly: 1.0085362). The foregoing computation may at first sight appear rather involved and awkward but will actually be found quite simple after some little practice, so that in time the computer will be able to make the corrections by visual estimate.

Corrections of the kind above reviewed are no longer necessary when the base drops below 1.001, because slide rule accuracy will then be equivalent to that obtainable by approximation.

Examples:

$1.021^{2.4} = 1.0512$	$8.5^{3.7} = 2750$	$.49^{2.8} = .1357$
$1.162^{-4} = .5485$	$25.4^{-2.6} = .000223$	$.49^{1.2} = .425$
$2.13^{5.3} = 55.0$	$e^{-\pi} = .0432$	$.49^{.12} = .918$
$2.13^{.53} = 1.493$	$.3^{4} = .0081$	$.93^{5.1} = .6906$
$2.13^{-.53} = .6698$	$.3^{-4} = 123$	$.93^{-5.1} = 1.448$

$1.19^{.31} = 1.0554$
$1.19^{.031} = 1.00538$ (approximation)
$1.19^{.031} = 1.00540$ (quadrature term applied)
$1.19^{.0031} = 1.000538$
$1.0048^{1.9} = 1.00912$ (approximation)
$1.0048^{1.9} = 1.00914$ (quadrature term applied)
$1.00021^{4.2} = 1.000883$

14.4 Powers $y = e^x$

When the indexes of the slide and the body scales are in coincidence the rule is adjusted to the equation $y = e^x$. The base $e = 2.718$ on scale LL3 being always aligned with the index of scale D, it follows that any power of e can be found by corresponding movement of the cursor to the exponent on scale D. The setting used in fig. 34, for instance, would be correct for the exponent 1.489 and its decimal variations:

$e^{1.489} = 4.43$	$e^{-1.489} = .2257$
$e^{.1489} = 1.1605$	$e^{-.1489} = .8617$
$e^{.01489} = 1.015$	$e^{-.01489} = .98522$

With $e^{.001489} = 1.001489$ we again arrive at the equivalence of $e^{\pm x} \approx 1 \pm x$.

14.5 Roots $a = \sqrt[x]{y}$

Expressions containing roots are often better to handle when changed to the form of a power. In this case set the exponent on CI.

$\sqrt[3.5]{e} = e^{\frac{1}{3.5}} = 1.3307$

$\dfrac{1}{\sqrt[.35]{e}} = e^{-\frac{1}{.35}} = .0574$

$\dfrac{1}{\sqrt[3.5]{e}} = e^{-\frac{1}{3.5}} = .7514$

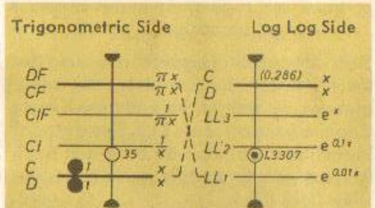


Fig. 39 Solution with the CI Scale

Inversely to the process of raising a number to a power we can also find the roots of numbers by using the Log Log scales in the same manner as the fundamental scales are used in division:

From $y = a^x$ we can derive $\sqrt[x]{y} = a$.

Procedure:

a. Set the radical index on C opposite the radicand y on scale LL.
b. Read the root under either the left or the right slide index on the appropriate Log Log scale.

The reading rules in chap. 14.2 are also in principle applicable in this instance. Bear in mind that when the reading is taken under the right slide index, the answer will appear on the next lower labeled Log Log scale LL1 – LL3 or LL01 – LL03.

$\sqrt[.77]{21} = 52.1$	$\dfrac{1}{\sqrt[.77]{21}} = .0192$
$\sqrt[7.7]{21} = 1.485$	$\dfrac{1}{\sqrt[7.7]{21}} = .6734$
$\sqrt[77]{21} = 1.0403$	$\dfrac{1}{\sqrt[77]{21}} = .96122$

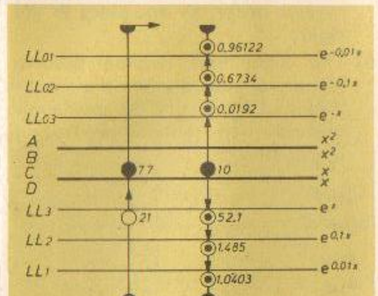


Fig. 40 Roots

14.6 Logarithms

With the Log Log scales logarithms to any base can be determined. By reversing the process of raising a number to a power, we obtain its logarithm:

$y = a^x \quad x = \log_a y$ (read: logarithm of y to the base a).

The finding of a logarithm is thus identical to a problem of powers in which the exponent is sought.

Procedure:

a. Set left end line of scale C over base value "a" on the appropriate Log Log scale.
b. Set hairline of cursor over the antilog y on the Log Log scale.
c. Read the logarithm under hairline of cursor on scale C.

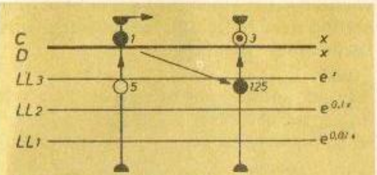


Fig. 41 $\log_5 125 = 3.0$

The decadal logarithms to the base 10 can be found in the same manner by setting the end line of scale C to the base on scale LL3. The decadal logarithms can also be obtained from the customary mantissa scale on the slide (Fig. 42).

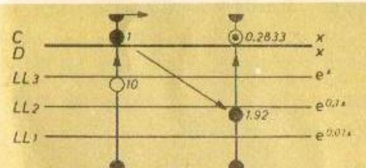


Fig. 42 $\log_{10} 1.920 = .2833$

The natural logarithms to the base e can be read directly on scale D (Fig. 43).

The position of the decimal point is derived from $\log_a a = 1$.

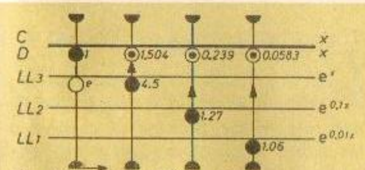


Fig. 43 $\log_e 1.06 = .0583$
$\log_e 1.27 = .239$
$\log_e 4.5 = 1.504$

With the index of the slide over the base "a" all values to the right of the value "a" on scale C are greater than 1 and all values to the left of "a" on scale C are smaller than 1.

Reading Rules:

a) Passing from one Log Log scale to the adjacent scale — in the order LL3, LL2, LL1 or LL03, LL02, LL01 — signifies a shift of the decimal point in the logarithm by one place to the left and, in the reverse order, by one place to the right.
b) The logarithms assume positive (negative) values when their antilogs and bases are set on equal-coloured (unequal-coloured) Log Log scales.

$\log_{10} 50 = 1.699$
$\log_{10} 2 = .3010$
$\log_{10} 1.03 = .01283$
$\log_{10} .015 = -1.824$
$\log_{10} .5 = -.3010$

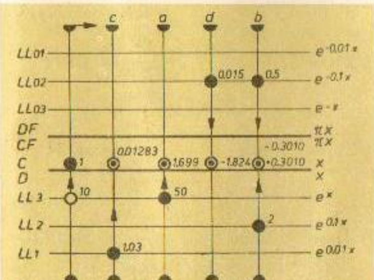


Fig. 44 Decadal Logarithms

When the slide is pushed out to the left of the body, all readings are taken from left of the base value. Since these values are < 1, the decimal point must logically be moved one place to the left as compared with the examples in Fig. 44.

Examples:

$\log_{10} 6 = .778$	$\log_2 16 = 4.0$	$\log_{2.5} 2 = -.5$
$\log_{10} 1.14 = .0569$	$\log_2 1.02 = .02857$	$\log_e .05 = -2.994$
$\log_{10} 1.015 = .00647$	$\log_2 .25 = -2$	$\log_e .622 = -.475$

15. Other Applications of the Log Log Scales

The slide of the Log Log side of the rule contains not only the fundamental scale C and the scale of squares B, but also the mantissa scale L and the scale of cubes K. Its usefulness is, therefore, not restricted to operations involving $x^2, x^3, \sqrt{x}, \sqrt[3]{x}$ and $\log_{10} x$ but is extended to powers of the form $a^{\sqrt{x}}, a^{\sqrt[3]{x}}, a^{10x}$ as well as, inversely, to logarithms of the form $\log_a^2 x, \log_a^3 x, \log_{10} \log_a x$.

The folded scales DF, CF and CIF can also be used in combination with the Log Log scales to avoid resetting the slide.

15.1 Solving Proportions with the Log Log Scales

When the slide index is set to some base value "a" on a Log Log scale, the powers to any exponent and also the logarithms of any number of this base can be obtained. The base a, when set on the Log Log scale, can therefore be regarded as one of the terms in a proportion.

15.1.1 $y_1 = a^n \qquad y_2 = a^m$

$\log y_1 = n \times \log a \quad \log y_2 = m \times \log a$

$$\frac{\log a}{1} = \frac{\log y_1}{n} = \frac{\log y_2}{m}$$

or $$\frac{\log_e a}{1} = \frac{\log_e y_1}{n} = \frac{\log_e y_2}{m}$$

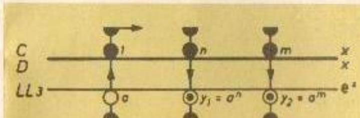


Fig. 45

When the given term of a proportion is set, each and every further term in the same ratio can be directly read.

Here we have another opportunity for applying the principle of proportion for which the slide rule is so well adapted.

15.1.2

$$y = a^{\frac{m}{n}} \rightarrow \log y = \frac{m}{n} \log a$$

$$\frac{\log y}{m} = \frac{\log a}{n}$$

$$y = 4.3^{\frac{6.8}{2.7}} \rightarrow \frac{\log y}{6.8} = \frac{\log 4.3}{2.7}$$

$y = 39.4$

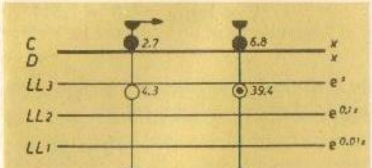


Fig. 46 $\frac{\log 4.3}{2.7} = \frac{\log 39.4}{6.8}$

After setting 2.7 on C opposite 4.3 on sca le LL3, the result 39.4 will be found on the LL3 scale under 6.8 of C. Modifications of this problem are, of course, solved analogously:

$$y = \sqrt[2.7]{4.3^{6.8}} \text{ or } y^{2.7} = 4.3^{6.8}$$

15.1.3

The formulas of many laws in natural sciences can be suitably arranged to permit a solution in the manner discussed above when the change in one variable is in proportion to the logarithm of the ratio of the other variable.

$$\log y_2/y_1 = \text{const} \times (x_2 - x_1)$$

Any mutation of x_1 to x_2 by the interval i entails a change of y_1 to y_2. When the ratio y_2/y_1 is given the designation r, i. e. the rest of the original whole quantity, the above equation can be written:

$$\frac{\log r}{i} = \text{const} = \frac{\log r_1}{i_1} = \frac{\log r_2}{i_2} = \cdots$$

Example: Radioactive Decay

A substance is known to disintegrate at the rate of 40% in 30 days (that is: leaving a residue of 60%).
$i_1 = 30$, $r_1 = .6$.
After how many days will 20% be left?
$r_2 = .2$

$$\frac{\log .6}{30} = \frac{\log .2}{x} \qquad x = 94.5 \text{ days}$$

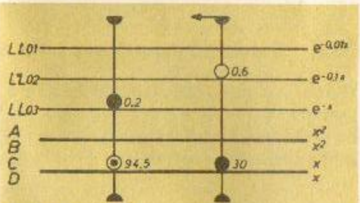


Fig. 47

15.1.4

For multiplication of a logarithm by a constant factor, the constant on C is set opposite to the base of the logarithm on the Log Log scale. Thus a tabulating position is obtained.

$x = c \times \log_a y$ or in proportion form

$$\frac{x}{\log_a y} = \frac{c}{1} = \frac{c}{{}^a\log a}$$

Examples: $2 \times \log_{10} 100 = 4$

$2 \times \log_{10} 1.8 = .511$

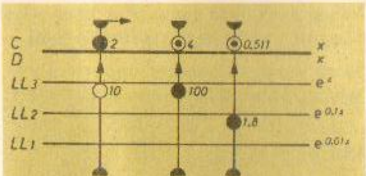


Fig. 48 $x = 2 \times \log_{10} y$

In electrical engineering it is often necessary to compute the decibel corresponding to a given voltage ratio:

$$db = 20 \times \log (U_1/U_2)$$

Logarithms to the base 10 can be multiplied with the factor 2 by the process illustrated in fig. 48, and with the LL_0 scales, also the logarithms of numbers smaller than 1.

15.2 Hyperbolic Functions

The unique construction of the Log Log scale system enables the formation of hyperbolic functions. The values of e^x and e^{-x} can be obtained by one setting of the cursor.

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

16. The Detachable Cursor and its Lines

16.1 The Mark 36 (Applies to 868 and 0986 only)

The front face of the cursor contains a short line to the right of the center line and running level with the folded sales. The lateral distance of this line from the center line corresponds to the factor 36 for readings on scale DF relative to any setting on the fundamental scale D. By virtue of this arrangement the cursor can be used for conversions of:

Years to Days:
1 year = 360 days
Hours to Seconds:
1 hour = 3600 seconds
1 meter per second = 3.6 kilometers per hour
Degrees to Seconds: 1° = 3600″

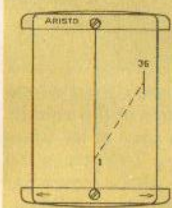


Fig. 49

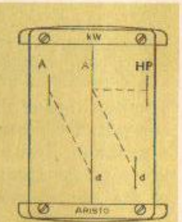


Fig. 50

16.2 The Marks for Circle Areas

The intervals between the upper left or the lower right line on the one hand and the center line on the other hand amount to $\pi/4 = .785$ i. e. the constant applicable in computations of circle areas or round sections $A = d^2 \times \pi/4$ (Fig. 50). To find any required circular area, set the lower right or the center hairline to the given diameter d on scale D and read the area under the center line or the upper left line, respectively, on scale A. With the 20″ model 1086 use the lower right and the upper left hair.

For users in countries where the metric system of weight and measurement is in force:

Since the specific weight of mild steel also happens to be 7.85 grams per cubic centimeter, it is easy to compute the weight of any running length of round rods. The short mark on the upper left of the cursor gives the weight in grams per cubic centimeter corresponding to the respective rod diameter in centimeters set with the lower right mark. The rest is then a simple multiplication. This is best done with the A and B scales by simply drawing the index of B under the left cursor hair and shifting the cursor to the given length on B.

Direct weight computations by cursor are not possible with the 20″ model No. 01068 because the cursor is not wide enough to cover twice the segment of scale corresponding to the factor $\pi/4$ on its expanded scales.

16.3 The Marks kW and HP

The interval between the upper right line and the center line is equivalent to the coefficient .746, applicable to conversions of HP to kW (Fig. 50). Hence, when the center hairline is set to 20 kW, for example, on the scale of squares, then the upper right line indicates the equivalent in HP viz. 26.8. Inversely, when the short right line is set to 7 HP the center line will produce the equivalent: 5.22 kW. On the 20″ model No. 01068 the kW and the HP mark are attached to the upper left and the upper right cursor hair, respectively.

16.4 Detaching the Cursor

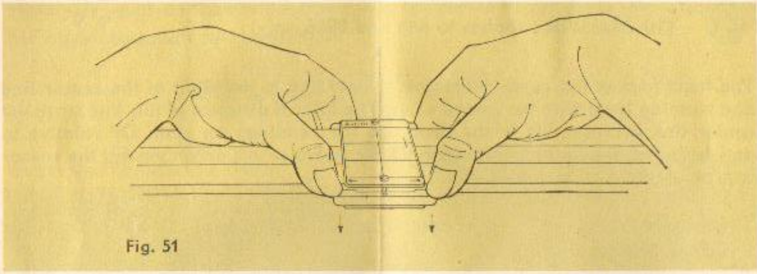


Fig. 51

The hairlines of the two cursor windows are precisely matched so that the user can pass from one face of the rule to the other when required in the course of a problem. The accuracy of its adjustment is not disturbed when the cursor is taken off for cleaning. To remove the cursor, use both thumbs to press the tips of the bar marked with arrows gently downwards. This releases the snap-fastener and the cursor can be taken off the slide rule (Fig. 51).

16.5 Adjustment of the Cursor

Even though the cursor hairs are reliably adjusted, violent jarring of the rule may throw them out of alignment. In such a case loosen the four screws on the cursor face with the HP mark. Turn the slide rule over and shift the other window about the center snap fastener until the hairline is accurately aligned to the index lines of the scales. Holding the adjusted window firmly in position, turn the slide rule over, and adjust the first window in a similar manner. Tighten all screws carefully to prevent renewed dislocation of the hairlines.

17. The ARISTO Conversion Table A

Probably the most important application of this comprehensive table of conversion factors is in connexion with the study of foreign technical literature employing units expressed in systems with which the reeder is not familiar. Metric versus non-metric standards are a case in point. Resort to reference books is time consuming and distracts the readers attention from the subject matter. The ARISTO Conversion table A serves as a ready reference covering the widest range of metric and non-metric equivalents, ratios between different denominations within the same system, conversion factors between power units and between energy units. Fits your slide rule case and can be used as a book mark.

18. Treatment of the ARISTO Slide Rule

The instrument is a valuable calculating aid and deserves careful treatment. Scales and cursor should be protected from dirt and scratches, so that the reading accuracy may not suffer.

It is advisable to give the rule an occasional treatment with the special cleanser fluid DEPAROL followed by a dry polishing. Avoid chemical substances of any description as they are almost certain to spoil the scales.

The slide rule must be protected from plastic erasers and their abrasive dusts, which can damage the surface of the material ARISTOPAL. Avoid also exposure to hot surfaces or bright sunshine, because at temperatures of about 140° F (60° C), distortion occurs. Rules so damaged will not be exchanged free of charge.

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